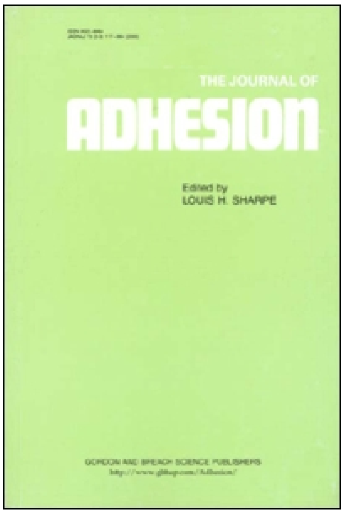

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Three-Dimensional Stress Analysis of Adhesive Butt Joints of Solid Cylinders Subjected to External Tensile Loads

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Three-Dimensional Stress Analysis of Adhesive Butt Joints of Solid Cylinders Subjected to External Tensile Loads

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This paper deals with a three-dimensional stress analysis of adhesive butt joints of similar solid cylinders subjected to external tensile loads. Similar adherends and an adhesive bond are replaced with finite solid cylinders in the analysis. Stress distributions on adhesive joints are analyzed strictly by using the three-dimensional theory of elasticity. The effects of stiffness, thickness and Poisson's ratio of the adhesive bonds and of external load distribution on the normal and the shear stress distributions are shown by numerical calculations. In addition, the analytical result is compared with the result obtained by F.E.M. It is seen that they are in fairly good agreement.

KEY WORDS Elasticity; stress analysis; adhesion; butt joint; solid cylinder; tensile load.

1 INTRODUCTION

Adhesive joints have been used in mechanical structures with the development of adhesion. However, the design of the adhesive joints results almost entirely from experience. It is hoped that this paper will make data available for design and will establish an optimal design method. In establishing the design method for adhesive joints, it is necessary to know the stress distributions in joints more precisely. Up to now, many investigations[1–7] have been carried out on lap, scarf and butt adhesive joints subjected to tensile, bending moment and shear loads. But few investigations[8,9] have been done on a butt joint in which two similar cylinders are joined by an adhesive.

This paper deals with a stress analysis of adhesive butt joints, in which two similar solid cylinders are joined by an adhesive and subjected to external tensile loads. Replacing similar adherends and the adhesive with finite solid cylinders, respectively, the stress distributions are analyzed strictly as a three-body contact problem by using the three-dimensional theory of elasticity. The effects of the ratio of Young's modulus and Poisson's ratio of the adherends to that of the adhesive, the distribution of external tensile loads and the thickness of the adhesives are clarified by numerical calculations. In addition, the analytical result is compared with the result obtained by F.E.M.[8]

2 THEORETICAL ANALYSIS

Figure 1 shows an adhesive butt joint, in which two similar finite solid cylinders are joined, subjected to an external axisymmetric tensile load. Taking into consideration the symmetry of the r-axis in the adhesive, the adherends are replaced with a finite solid cylinder [I] and the adhesive with a finite solid cylinder [II]. The diameter of cylinder [I] is designated by $2a$, the height by $2h_1$, Young's modulus by E_1 and Poisson's ratio by ν_1. The same parameters for cylinder [III] are designated by $2a$, $2h_2$, E_2 and ν_2, respectively. It is assumed that an external tensile load $F(r)$ acts on the ends of cylinder [I] within the region $r<c$ axisymmetrically with respect to the z_1-axis. Developing the load distribution $F(r)$ into series of Bessel functions, the boundary conditions are expressed as follows:

(i) on the cylinder [I] (adherends)

$$\begin{aligned} r = a &: \sigma_r^{\mathrm{I}} = \tau_{rz}^{\mathrm{I}} = 0 \\ z_1 = h_1 &: \sigma_z^{\mathrm{I}} = F(r) = a_0 + \sum_{s=1}^{\infty} a_s J_0(\gamma_s r) \\ \tau_{zr}^{\mathrm{I}} &= 0 \end{aligned} \tag{1}$$

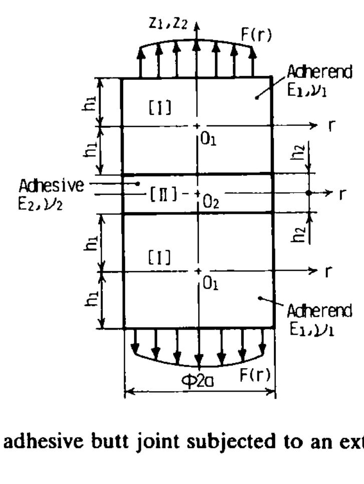

FIGURE 1 An adhesive butt joint subjected to an external tensile load.

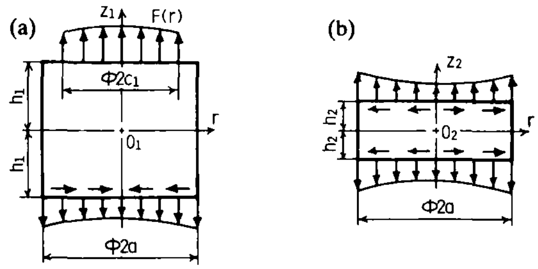

FIGURE 2 A model for analysis and dimensions. (a) finite solid cylinder [I] (adherend) (b) finite solid cylinder [II] (adhesive).

(ii) on the cylinder [II] (adhesive)

$$r = a : \sigma_r^{II} = \tau_{rz}^{II} = 0 \tag{2}$$

(iii) at the interface

$$\begin{aligned} (\sigma_z^{I})_{z_1=-h_1} &= (\sigma_z^{II})_{z_2=h_2} \\ (\tau_{zr}^{I})_{z_1=-h_1} &= (\tau_{zr}^{II})_{z_2=h_2} \\ (u^{I})_{z_1=-h_1} &= (u^{II})_{z_2=h_2} \\ \left(\frac{\partial w^{I}}{\partial r}\right)_{z_1=-h_1} &= \left(\frac{\partial w^{II}}{\partial r}\right)_{z_2=h_2} \end{aligned} \tag{3}$$

where the displacement in the r-direction is denoted by u and the displacement in the z-direction by w, and

$$a_0 = \frac{2}{a^2}\int_0^a F(r) r\, dr$$

$$a_s = \frac{2}{a^2 J_0^2(\gamma_s a)}\int_0^a F(r) r J_0(\gamma_s r)\, dr \quad (s = 1, 2, 3, \ldots)$$

Since the different coordinates 0_1 and 0_2 are fixed in the analysis, the boundary condition

$$\left(\frac{\partial w^{I}}{\partial r}\right)_{z=-h_1} = \left(\frac{\partial w^{II}}{\partial r}\right)_{z=h_2}$$

in Eq. (2) is used. In the analysis of the finite solid cylinders under the boundary conditions, Eqs. (1)–(3), Michell's stress function Φ is used. Using the stress function Φ, the stresses and displacements are given as follows[10]:

$$\begin{aligned} \sigma_r &= \frac{\partial}{\partial z}\left(\nu\nabla^2\Phi - \frac{\partial^2\Phi}{\partial r^2}\right) \\ \sigma_\theta &= \frac{\partial}{\partial z}\left(\nu\nabla^2\Phi - \frac{1}{r}\frac{\partial\Phi}{\partial r}\right) \\ \sigma_z &= \frac{\partial}{\partial z}\left\{(2-\nu)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right\} \\ \tau_{rz} &= \frac{\partial}{\partial r}\left\{(1-\nu)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right\} \end{aligned} \tag{4}$$

$$\begin{aligned} u &= -\frac{1+\nu}{E}\frac{\partial^2\Phi}{\partial r\,\partial z} \\ w &= \frac{1+\nu}{E}\left\{(1-2\nu)\nabla^2\Phi + \frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r}\right\} \end{aligned} \tag{5}$$

where

$$\nabla^2\nabla^2\Phi = 0 \quad \left(\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right),$$

and E is Young's modulus and ν is Poisson's ratio.

Michell's stress function Φ^{I}, which is selected from solutions for the method of separation of variables, is put as $\Phi_1^{\mathrm{I}} + \Phi_2^{\mathrm{I}} + \Phi_3^{\mathrm{I}} + \Phi_4^{\mathrm{I}}$ for cylinder [I] and Φ^{II} as $\Phi_1^{\mathrm{II}} + \Phi_3^{\mathrm{II}}$ for cylinder [II]. The stress functions Φ_1^{I}, Φ_2^{I}, Φ_3^{I}, Φ_4^{I}, Φ_1^{II} and Φ_3^{II} are expressed by Eqs. (6)–(11), where $\bar{A}_0^{\mathrm{I}}$, $\bar{C}_0^{\mathrm{I}}$, $\bar{A}_n^{\mathrm{I}}$, $\bar{C}_s^{\mathrm{I}}$, $\bar{\bar{A}}_n^{\mathrm{I}}$, $\bar{\bar{C}}_s^{\mathrm{I}}$, $\tilde{A}_0^{\mathrm{I}}$, $\tilde{C}_0^{\mathrm{I}}$, $\tilde{A}_n^{\mathrm{I}}$, $\tilde{C}_s^{\mathrm{I}}$, $\tilde{\bar{A}}_n^{\mathrm{I}}$, $\tilde{\bar{C}}_s^{\mathrm{I}}$, $\bar{A}_0^{\mathrm{II}}$, $\bar{C}_0^{\mathrm{II}}$, $\bar{A}_n^{\mathrm{II}}$, $\bar{C}_s^{\mathrm{II}}$, $\tilde{A}_0^{\mathrm{II}}$, $\tilde{C}_0^{\mathrm{II}}$, $\tilde{A}_n^{\mathrm{II}}$, and $\tilde{C}_s^{\mathrm{II}}$ $(n, s = 1, 2, 3, \ldots)$ are unknown coefficients determined from the boundary conditions.

$$\begin{aligned}
\Phi_1^{\mathrm{I}} &= \Phi_1(\bar{A}_0^{\mathrm{I}}, \bar{C}_0^{\mathrm{I}}, \bar{A}_n^{\mathrm{I}}, \bar{C}_s^{\mathrm{I}}\, a, h_1, \beta_n^{\mathrm{I}}, \gamma_s, \bar{\Delta}_n^{\mathrm{I}}, \bar{\Omega}_s^{\mathrm{I}}, \nu_1, r, z_1)\\
&= \bar{A}_0^{\mathrm{I}}\frac{z_1^3}{6} + \bar{C}_0^{\mathrm{I}}\frac{z_1 r^2}{2} + \sum_{n=1}^{\infty}\frac{\bar{A}_n^{\mathrm{I}}}{\beta_n^{\mathrm{I}3}\bar{\Delta}_n^{\mathrm{I}}}\\
&\quad\times[\{2(1-\nu_1)I_{1a} + \beta_n^{\mathrm{I}} a I_{0a}\}I_{0r} - \beta_n^{\mathrm{I}} r I_{1a} I_{1r}]\sin(\beta_n^{\mathrm{I}} z_1)\\
&\quad+\sum_{s=1}^{\infty}\frac{\bar{C}_s^{\mathrm{I}}}{\gamma_s^3\bar{\Omega}_s^{\mathrm{I}}}[-\{2\nu_1\,\mathrm{sh}(\gamma_s h_1) + \gamma_s h_1\,\mathrm{ch}(\gamma_s h_1)\}\\
&\quad\times\mathrm{sh}(\gamma_s z_1) + \gamma_s z_1\,\mathrm{sh}(\gamma_s h_1)\mathrm{ch}(\gamma_s z_1)]J_0(\gamma_s r)
\end{aligned}\tag{6}$$

$$\begin{aligned}
\Phi_2^{\mathrm{I}} &= \Phi_2(\bar{\bar{A}}_n^{\mathrm{I}}, \bar{\bar{C}}_s^{\mathrm{I}}, a, h_1, \beta_n^{\mathrm{I}'}, \gamma_s, \bar{\bar{\Delta}}_n^{\mathrm{I}}, \bar{\bar{\Omega}}_s^{\mathrm{I}}, \nu_1, r, z_1)\\
&= -\sum_{n=1}^{\infty}\frac{\bar{\bar{A}}_n^{\mathrm{I}}}{\beta_n^{\mathrm{I}'3}\bar{\bar{\Delta}}_n^{\mathrm{I}}}[\{2(1-\nu_1)I'_{1a} + \beta_n^{\mathrm{I}'} a I'_{0a}\}\\
&\quad\times I'_{0r} - \beta_n^{\mathrm{I}'} r I'_{1a} I'_{1r}]\cos(\beta_n^{\mathrm{I}'} z_1)\\
&\quad+\sum_{s=1}^{\infty}\frac{\bar{\bar{C}}_s^{\mathrm{I}}}{\gamma_s^3\bar{\bar{\Omega}}_s^{\mathrm{I}}}[-\{2\nu_1\,\mathrm{ch}(\gamma_s h_1) + \gamma_s h_1\,\mathrm{sh}(\gamma_s h_1)\}\\
&\quad\times\mathrm{ch}(\gamma_s z_1) + \gamma_s z_1\,\mathrm{ch}(\gamma_s h_1)\mathrm{sh}(\gamma_s z_1)]J_0(\gamma_s r)
\end{aligned}\tag{7}$$

$$\begin{aligned}
\Phi_3^{\mathrm{I}} &= \Phi_3(\tilde{A}_0^{\mathrm{I}}, \tilde{C}_0^{\mathrm{I}}, \tilde{A}_n^{\mathrm{I}}, \tilde{C}_s^{\mathrm{I}}, a, h_1, \beta_n^{\mathrm{I}'}, \gamma_s, \tilde{\Delta}_n^{\mathrm{I}}, \tilde{\Omega}_s^{\mathrm{I}}, \nu_1, r, z_1)\\
&= \tilde{A}_0^{\mathrm{I}}\frac{z_1^3}{6} + \tilde{C}_0^{\mathrm{I}}\frac{z_1 r^2}{2} + \sum_{n=1}^{\infty}\frac{\tilde{A}_n^{\mathrm{I}}}{\beta_n^{\mathrm{I}'3}\tilde{\Delta}_n^{\mathrm{I}}}\\
&\quad\times[\{2(1-\nu_1)I'_{1a} + \beta_n^{\mathrm{I}'} a I'_{0a}\}I'_{0r} - \beta_n^{\mathrm{I}'} r I'_{1a} I'_{1r}]\sin(\beta_n^{\mathrm{I}'} z_1)\\
&\quad+\sum_{s=1}^{\infty}\frac{\tilde{C}_s^{\mathrm{I}}}{\gamma_s^3\tilde{\Omega}_s^{\mathrm{I}}}[\{(1-2\nu_1)\mathrm{ch}(\gamma_s h_1) - \gamma_s h_1\,\mathrm{sh}(\gamma_s h_1)\}\\
&\quad\times\mathrm{sh}(\gamma_s z_1) + \gamma_s z_1\,\mathrm{ch}(\gamma_s h_1)\mathrm{ch}(\gamma_s z_1)]J_0(\gamma_s r)
\end{aligned}\tag{8}$$

$$\begin{aligned}
\Phi_4^{\mathrm{I}} &= \Phi_4(\tilde{\bar{A}}_n^{\mathrm{I}}, \tilde{\bar{C}}_s^{\mathrm{I}}, a, h_1, \beta_n^{\mathrm{I}}, \gamma_s, \tilde{\bar{\Delta}}_n^{\mathrm{I}}, \tilde{\bar{\Omega}}_s^{\mathrm{I}}, \nu_1, r, z_1)\\
&= -\sum_{n=1}^{\infty}\frac{\tilde{\bar{A}}_n^{\mathrm{I}}}{\beta_n^{\mathrm{I}3}\tilde{\bar{A}}_n^{\mathrm{I}}}[\{2(1-\nu_1)I_{1a} + \beta_n^{\mathrm{I}} a I_{0a}\}I_{0a} - \beta_n^{\mathrm{I}} r I_{1a} I_{1r}]\cos(\beta_n^{\mathrm{I}} z_1)\\
&\quad+\sum_{s=1}^{\infty}\frac{\tilde{\bar{C}}_s^{\mathrm{I}}}{\gamma_s^3\tilde{\bar{\Omega}}_s^{\mathrm{I}}}[\{(1-2\nu_1)\mathrm{sh}(\gamma_s h_1) - \gamma_s h_1\,\mathrm{ch}(\gamma_s h_1)\}\\
&\quad\times\mathrm{ch}(\gamma_s z_1) + \gamma_s z_1\,\mathrm{sh}(\gamma_s h_1)\mathrm{sh}(\gamma_s z_1)]J_0(\gamma_s r)
\end{aligned}\tag{9}$$

$$\Phi_1^{II} = \Phi_1(\bar{A}_0^{II}, \bar{C}_0^{II}, \bar{A}_n^{II}, \bar{C}_s^{II}, a, h_2, \beta_n^{II}, \gamma_s, \bar{\Delta}_n^{II}, \bar{\Omega}_s^{II}, \nu_2, r, z_2) \tag{10}$$

$$\Phi_3^{II} = \Phi_3(\bar{A}_0^{II}, \bar{C}_0^{II}, \bar{A}_n^{II}, \bar{C}_s^{II}, a, h_2, \beta_n^{II'}, \gamma_s, \bar{\Delta}_n^{II}, \bar{\Omega}_s^{II}, \nu_2, r, z_2) \tag{11}$$

where $J_\mu(r)$ is the first kind of Bessel function of order μ, $I_\mu(r)$ is the first kind of modified Bessel function of order μ and λ_s is the positive root satisfying the equation $J_1(\lambda_s) = 0$, and $I_\mu(\beta_n a)$ is abbreviated as $I_{\mu a}$, $I_\mu(\beta_n r)$ as $I_{\mu r}$, $I_\mu(\beta_n' a)$ as $I'_{\mu a}$, $I_\mu(\beta_n' r)$ as $I'_{\mu r}$, and sinh is abbreviated as sh and cosh as ch.

$$\beta_n^I = \beta_n(h_1) = \frac{n\pi}{h_1}, \quad \beta_n^{I'} = \beta_n'(h_1) = \frac{(2n-1)\pi}{2h_1}, \quad \beta_n^{II} = \beta_n(h_2), \quad \beta_n^{II'} = \beta_n'(h_2)$$

$$\gamma_s = \lambda_s/a, \quad \bar{\Omega}_s^I = \Omega_s(\gamma_s h_1) = \mathrm{sh}(\gamma_s h_1)\mathrm{ch}(\gamma_s h_1) + \gamma_s h_1$$

$$\bar{\bar{\Omega}}_s^I = \Omega_s'(\gamma_s h_1) = \mathrm{sh}(\gamma_s h_1)\mathrm{ch}(\gamma_s h_1) - \gamma_s h_1, \quad \tilde{\Omega}_s^I = \bar{\Omega}_s^I, \quad \tilde{\bar{\Omega}}_s^I = \bar{\bar{\Omega}}_s^I$$

$$\bar{\Omega}_s^{II} = \Omega_s(\gamma_s h_2), \quad \tilde{\Omega}_s^{II} = \bar{\Omega}_s^{II}$$

$$\bar{\Delta}_n^I = \Delta_n(\beta_n^I a, \nu_1) = [\{2(1-\nu_1) + (\beta_n^I a)^2\} I_{1a}^2 - (\beta_n^I a) I_{0a}^2]/\beta_n^I a$$

$$\bar{\bar{\Delta}}_n^I = \Delta_n(\beta_n^{I'} a, \nu_1), \quad \tilde{\Delta}_n^I = \bar{\bar{\Delta}}_n^I, \quad \tilde{\bar{\Delta}}_n^I = \bar{\Delta}_n^I$$

$$\bar{\Delta}_n^{II} = \Delta_n(\beta_n^{II} a, \nu_2), \quad \tilde{\Delta}_n^{II} = \Delta_n(\beta_n^{II'} a, \nu_2)$$

Substituting Eqs. (6)–(11) into Eqs. (4) and (5), the stresses and displacements are obtained. Conforming the obtained stresses and displacements to the boundary conditions (1)–(3), the following equations (12) and (13) among the unknown coefficients are obtained.

$$\bar{A}_0^I(1-\nu_1) + \bar{C}_0^I 2(2-\nu_1) = a_0, \quad \bar{A}_0^I \nu + \bar{C}_0^I(2\nu_1 - 1) = 0,$$

$$\tilde{A}_0^I(1-\nu_1) + \tilde{C}_0^I 2(2-\nu_1) = 0$$

$$\tilde{A}_0^I \nu_1 + \tilde{C}_0^I(2\nu_1 - 1) + \sum_{n=1}^{\infty} \tilde{A}_n^I \frac{(-1)^{n+1}}{\beta_n^{I'} h_1} + \sum_{s=1}^{\infty} \tilde{C}_s^I \frac{J_0(\gamma_s a)}{\gamma_s h_1} = 0$$

$$\bar{A}_0^{II}(1-\nu_2) + \bar{C}_0^{II} 2(2-\nu_2) = a_0, \quad \bar{A}_0^{II}\nu_2 + \bar{C}_0^{II}(2\nu_2 - 1) = 0,$$

$$\tilde{A}_0^{II}(1-\nu_2) + \tilde{C}_0^{II} 2(2-\nu_2) = 0$$

$$\tilde{A}_0^{II}\nu_2 + \tilde{C}_0^{II}(2\nu - 1) + \sum_{n=1}^{\infty} \tilde{A}_n^{II} \frac{(-1)^{n+1}}{\beta_n^{II'} h_2} + \sum_{s=1}^{\infty} \tilde{C}_s^{II} \frac{J_0(\gamma_s a)}{\gamma_s h_2} = 0 \tag{12}$$

$$\bar{A}_n^I + \sum_{s=1}^{\infty} \bar{C}_s^I \bar{Q}_{sn}^I = 0, \quad \bar{\bar{A}}_n^I + \sum_{s=1}^{\infty} \bar{\bar{C}}_s^I \bar{\bar{Q}}_{sn}^I = 0,$$

$$\sum_{n=1}^{\infty} \tilde{A}_n^I \tilde{S}_{nm}^I + \sum_{s=1}^{\infty} \tilde{C}_s^I \tilde{T}_{sm}^I = 0, \quad \tilde{\bar{A}}_n^I + \sum_{s=1}^{\infty} \tilde{\bar{C}}_s^I \tilde{\bar{Q}}_{sm}^I = 0$$

$$\bar{A}_n^{II} + \sum_{s=1}^{\infty} \bar{C}_s^{II} \bar{Q}_{sn}^{II} = 0, \quad \sum_{n=1}^{\infty} \tilde{A}_n^{II} \tilde{S}_{nm}^{II} + \sum_{s=1}^{\infty} \tilde{C}_s^{II} \tilde{T}_{sm}^{II} = 0,$$

$$\sum_{n=1}^{\infty} \bar{A}_n^I \bar{R}_{ns}^I + \bar{C}_s^I - \sum_{n=1}^{\infty} \bar{\bar{A}}_n^I \bar{\bar{R}}_{ns}^I + \bar{\bar{C}}_s^I = a_s$$

$$\sum_{n=1}^{\infty} \tilde{A}_n^I \tilde{R}_{ns}^I + \tilde{C}_s^I + \sum_{n=1}^{\infty} \tilde{\bar{A}}_n^I \tilde{\bar{R}}_{ns}^I + \tilde{\bar{C}}_s^I = 0,$$

$$\sum_{n=1}^{\infty} \bar{A}_n^I \bar{R}_{ns}^I + \bar{C}_s^I + \sum_{n=1}^{\infty} \bar{\bar{A}}_n^I \bar{\bar{R}}_{ns}^I - \bar{\bar{C}}_s^I - \sum_{n=1}^{\infty} \bar{A}_n^{II} \bar{R}_{ns}^{II} - \bar{C}_s^{II} = 0$$

$$\sum_{n=1}^{\infty} \bar{A}_n^{\mathrm{I}} \bar{R}_{ns}^{\mathrm{I}} + \bar{C}_s^{\mathrm{I}} - \sum_{n=1}^{\infty} \bar{\bar{A}}_n^{\mathrm{I}} \bar{\bar{R}}_{ns}^{\mathrm{I}} - \bar{\bar{C}}_s^{\mathrm{I}} + \sum_{n=1}^{\infty} \bar{A}_n^{\mathrm{II}} \bar{R}_{ns}^{\mathrm{II}} + C_s^{\mathrm{II}} = 0$$

$$\sum_{n=1}^{\infty} \bar{A}_n^{\mathrm{I}} \bar{H}_{ns}^{\mathrm{I}} - \bar{C}_s^{\mathrm{I}} \bar{F}_s^{\mathrm{I}} + \sum_{n=1}^{\infty} \bar{\bar{A}}_n^{\mathrm{I}} \bar{\bar{H}}_{ns}^{\mathrm{I}} + \bar{\bar{C}}_s^{\mathrm{I}} \bar{\bar{F}}_s^{\mathrm{I}} - \bar{C}_s^{\mathrm{I}} \bar{J}_s^{\mathrm{I}} + \bar{\bar{C}}_s^{\mathrm{I}} \bar{\bar{J}}_s^{\mathrm{I}}$$
$$- \sum_{n=1}^{\infty} \bar{A}_n^{\mathrm{II}} \bar{H}_{ns}^{\mathrm{II}} + \bar{C}_s^{\mathrm{II}} \bar{F}_s^{\mathrm{II}} - \bar{C}_s^{\mathrm{II}} \bar{J}_s^{\mathrm{II}} = D_s$$

$$\bar{C}_s^{\mathrm{I}} \bar{U}_s^{\mathrm{I}} - \bar{\bar{C}}_s^{\mathrm{I}} \bar{\bar{U}}_s^{\mathrm{I}} - \sum_{n=1}^{\infty} \bar{A}_n^{\mathrm{I}} \bar{P}_{ns}^{\mathrm{I}} + \bar{C}_s^{\mathrm{I}} \bar{W}_s^{\mathrm{I}} + \sum_{n=1}^{\infty} \bar{\bar{A}}_n^{\mathrm{I}} \bar{\bar{P}}_{ns}^{\mathrm{I}} - \bar{\bar{C}}_s^{\mathrm{I}} \bar{\bar{W}}_s^{\mathrm{I}} + \bar{C}_s^{\mathrm{II}} \bar{U}_s^{\mathrm{II}}$$
$$- \sum_{n=1}^{\infty} \bar{A}_n^{\mathrm{II}} \bar{P}_{ns}^{\mathrm{II}} + \bar{C}_s^{\mathrm{II}} \bar{W}_s^{\mathrm{II}} = 0 \quad (13)$$

where

$$\bar{Q}_{sn}^{\mathrm{I}} = Q_{sn}(h_1, \bar{\Omega}_s^{\mathrm{I}}) = \frac{4(-1)^n \gamma_s h_1 (n\pi)^2 J_0(\gamma_s a) \mathrm{sh}^2(\gamma_s h_1)}{\bar{\Omega}_s^{\mathrm{I}} \{(\gamma_s h_1)^2 + (n\pi)^2\}^2},$$

$$\bar{\bar{Q}}_{sn}^{\mathrm{I}} = \frac{(-1)^{n+1} \gamma_s h_1 (2n-1)^2 \pi^2 J_0(\gamma_s a) \mathrm{ch}^2(\gamma_s h_1)}{\bar{\bar{\Omega}}_s^{\mathrm{I}} [(\gamma_s h_1)^2 + \{(2n-1)\pi/2\}^2]^2}$$

$$\bar{\bar{Q}}_{sn}^{\mathrm{I}} = \frac{4(-1)^{n+1} (n\pi)^3 J_0(\gamma_s a) \mathrm{sh}^2(\gamma_s h_1)}{\bar{\Omega}_s^{\mathrm{I}} \{(\gamma_s h_1)^2 + (n\pi)^2\}^2}, \quad \bar{Q}_{sn}^{\mathrm{II}} = Q_{sn}(h_2, \bar{\Omega}_s^{\mathrm{II}})$$

$$\bar{S}_{nm}^{\mathrm{I}} = S_{nm}(\beta_n^{\mathrm{I}'}, \beta_m^{\mathrm{I}}) = (-1)^{m+n+1} \{1/(\beta_n^{\mathrm{I}'} + \beta_m^{\mathrm{I}}) + 1/(\beta_n^{\mathrm{I}'} - \beta_m^{\mathrm{I}})\},$$

$$\bar{S}_{nm}^{\mathrm{II}} = S_{nm}(\beta_n^{\mathrm{II}'}, \beta_m^{\mathrm{II}})$$

$$\bar{T}_{sm}^{\mathrm{I}} = T_{sm}(h_1, \beta_m^{\mathrm{I}}, \bar{\Omega}_s^{\mathrm{I}}) = \frac{2(-1)^m \gamma_s J_0(\gamma_s a)}{\bar{\Omega}_s^{\mathrm{I}} (\gamma_s^2 + \beta_m^{\mathrm{I}^2})} \left\{ \gamma_s h_1 + \frac{\gamma_s^2 + 3\beta_m^{\mathrm{I}^2}}{\gamma_s^2 + \beta_m^{\mathrm{I}^2}} \mathrm{sh}(\gamma_s h_1) \mathrm{ch}(\gamma_s h_1) \right\}$$

$$\bar{T}_{sm}^{\mathrm{II}} = T_{sm}(h_2, \beta_m^{\mathrm{II}}, \bar{\Omega}_s^{\mathrm{II}}), \quad \bar{R}_{ns}^{\mathrm{I}} = R_{ns}(\beta_n^{\mathrm{I}}, \bar{\Delta}_n^{\mathrm{I}}) = \frac{4(-1)^{n+1} \lambda_s^2 \beta_n^{\mathrm{I}} a I_{1a}^2}{\bar{\Delta}_n^{\mathrm{I}} \{(\beta_n^{\mathrm{I}} a)^2 + \lambda_s^2\}^2 J_0(\gamma_s a)}$$

$$\bar{\bar{R}}_{ns}^{\mathrm{I}} = R_{ns}(\beta_n^{\mathrm{I}'}, \bar{\bar{\Delta}}_n^{\mathrm{I}}), \quad \bar{R}_{ns}^{\mathrm{I}} = R'_{ns}(\beta_n^{\mathrm{I}'}, \bar{\Delta}_n^{\mathrm{I}}) = \frac{4(-1)^n \lambda_s (\beta_n^{\mathrm{I}'} a)^2 I_{1a}'^2}{\bar{\Delta}_n^{\mathrm{I}} \{(\beta_n^{\mathrm{I}'} a)^2 + \lambda_s^2\}^2 J_0(\gamma_s a)}$$

$$\bar{\bar{R}}_{ns}^{\mathrm{I}} = R'_{ns}(\beta_n^{\mathrm{I}}, \bar{\bar{\Delta}}_n^{\mathrm{I}}), \quad \bar{R}_{ns}^{\mathrm{II}} = R_{ns}(\beta_n^{\mathrm{II}}, \bar{\Delta}_n^{\mathrm{II}}), \quad \bar{R}_{ns}^{\mathrm{II}} = R'_{ns}(\beta_n^{\mathrm{II}'}, \bar{\Delta}_n^{\mathrm{II}})$$

$$\bar{H}_{ns}^{\mathrm{I}} = H_{ns}(\beta_n^{\mathrm{I}}, \bar{\Delta}_n^{\mathrm{I}}, G_1, \nu_1) = \frac{4(-1)^{n+1} \gamma_s I_{1a}^2}{\bar{\Delta}_n^{\mathrm{I}} \beta_n^{\mathrm{I}} a J_0(\gamma_s a)(\gamma_s^2 + \beta_n^{\mathrm{I}^2}) G_1} \left(1 - \nu_1 + \frac{\beta_n^{\mathrm{I}^2}}{\gamma_s^2 + \beta_n^{\mathrm{I}^2}} \right),$$

$$\bar{\bar{H}}_{ns}^{\mathrm{I}} = H_{ns}(\beta_n^{\mathrm{I}'}, \bar{\bar{\Delta}}_n^{\mathrm{I}}, G_1, \gamma_1)$$

$$\bar{H}_{ns}^{\mathrm{II}} = H_{ns}(\beta_n^{\mathrm{II}}, \bar{\Delta}_n^{\mathrm{II}}, G_2, \nu_2),$$

$$\bar{F}_s^{\mathrm{I}} = F_s(h_1, \bar{\Omega}_s^{\mathrm{I}}, G_1, \nu_1) = \frac{1}{\gamma_s \bar{\Omega}_s^{\mathrm{I}} G_1} \{(1 - 2\nu_1) \mathrm{sh}(\gamma_s h_1) \mathrm{ch}(\gamma_s h_1) - \gamma_s h_1\}$$

$$\bar{\bar{F}}_s^{\mathrm{I}} = F'_s(h_1, \bar{\bar{\Omega}}_s^{\mathrm{I}}, G_1, \nu_1) = \frac{1}{\gamma_s \bar{\bar{\Omega}}_s^{\mathrm{I}} G_1} \{(1 - 2\nu_1) \mathrm{sh}(\gamma_s h_1) \mathrm{ch}(\gamma_s h_1) + \gamma_s h_1\},$$

$$\bar{F}_s^{\mathrm{II}} = F_s(h_2, \bar{\Omega}_s^{\mathrm{II}}, G_2, \nu_2)$$

$$\bar{J}_s^{\mathrm{I}}=\frac{2(1-\nu_1)\mathrm{ch}^2(\gamma_s h_1)}{\gamma_s\bar{\Omega}_s^{\mathrm{I}}G_1},\quad \bar{\bar{J}}_s^{\mathrm{I}}=\frac{2(1-\nu_1)\mathrm{sh}^2(\gamma_s h_1)}{\gamma_s\bar{\bar{\Omega}}_s^{\mathrm{I}}G_1},\quad \bar{J}_s^{\mathrm{II}}=\frac{2(1-\nu_2)\mathrm{ch}^2(\gamma_s h_2)}{\gamma_s\bar{\Omega}_s^{\mathrm{II}}G_2}$$

$$D_s=\frac{2}{\gamma_s J_0(\gamma_s a)}\left(\frac{\bar{C}_0^{\mathrm{I}}}{G_1}+\frac{\bar{\bar{C}}_0^{\mathrm{I}}}{G_1}-\frac{\bar{C}_0^{\mathrm{II}}}{G_2}-\frac{\bar{\bar{C}}_0^{\mathrm{II}}}{G_2}\right),$$

$$\bar{U}_s^{\mathrm{I}}=\frac{2(1-\nu_1)\mathrm{sh}^2(\gamma_s h_1)}{\bar{\Omega}_s^{\mathrm{I}}G_1},\quad \bar{\bar{U}}_s^{\mathrm{I}}=\frac{2(1-\nu_1)\mathrm{ch}^2(\gamma_s h_1)}{\bar{\bar{\Omega}}_s^{\mathrm{I}}G_1}$$

$$\bar{U}_s^{\mathrm{II}}=\frac{2(1-\nu_2)\mathrm{sh}^2(\gamma_s h_2)}{\bar{\Omega}_s^{\mathrm{II}}G_2},\quad \bar{W}_s^{\mathrm{I}}=\gamma_s\times F_s(h_1,\bar{\Omega}_s^{\mathrm{I}},G_1,\nu_1),$$

$$\bar{\bar{W}}_s^{\mathrm{I}}=\gamma_s\times F_s'(h_1,\bar{\bar{\Omega}}_s^{\mathrm{I}},G_1,\nu_1)$$

$$\bar{W}_s^{\mathrm{II}}=\gamma_s\times F_s(h_2,\bar{\Omega}_s^{\mathrm{II}},G_2,\nu_2),$$

$$\bar{P}_{ns}^{\mathrm{I}}=\bar{P}_{ns}(\beta_n^{\mathrm{I}'},\bar{\Delta}_n^{\mathrm{I}},G_1,\nu_1)=\frac{4(-1)^n\gamma_s I_{1a}'^2}{\bar{\Delta}_n^{\mathrm{I}}aJ_0(\gamma_s a)(\gamma_s^2+\beta_n^{\mathrm{I}'^2})G_1}\left(\nu_1-1+\frac{\beta_n^{\mathrm{I}'^2}}{\gamma_s^2+\beta_n^{\mathrm{I}'^2}}\right)$$

$$\bar{\bar{P}}_{ns}^{\mathrm{I}}=P_{ns}(\beta_n^{\mathrm{I}},\bar{\bar{\Delta}}_n^{\mathrm{I}},G_1,\nu_1),\quad \bar{P}_{ns}^{\mathrm{II}}=P_{ns}(\beta_n^{\mathrm{II}'},\bar{\Delta}_n^{\mathrm{II}},G_2,\nu_2),$$

$$G_1=E_1/2(1+\nu_1),\quad G_2=E_2/2(1+\nu_2)$$

Putting the number of terms as N in numerical computations and solving $12N$ simultaneous equations of Eq. (13), the unknown coefficients $\bar{A}_n^{\mathrm{I}}, \bar{C}_s^{\mathrm{I}}, \ldots, \bar{A}_n^{\mathrm{II}}$ and $\bar{C}_s^{\mathrm{II}}$ are determined. In addition, the unknown coefficients $\bar{A}_0^{\mathrm{I}}, \bar{C}_0^{\mathrm{I}}, \ldots, \bar{A}_0^{\mathrm{II}}$ and $\bar{C}_0^{\mathrm{II}}$ are determined from Eq. (12). Using the above determined coefficients, the stresses and displacements are obtained.

3 RESULTS AND DISCUSSION

In the present analysis, the stress singularity at the edge of the interfaces is not taken into consideration. Hence, computations were done varying the number N of terms as 100 and 120 in order to examine the effect of the number N on the stress distributions at the interfaces. As a result, it was seen that the difference between both results was less than 3%. Hereafter, computations were done setting the number N of terms as 100. Figure 3 shows the effect of the ratio E_1/E_2 of Young's modulus of the adherends to that of the adhesive on the stress distributions at the interface ($z_2=h_2$), where the values E_1/E_2 were varied as 1, 3, 40, 60 and infinity. In the case where the value E_1/E_2 was infinity, that is the adherends were assumed rigid, the analysis was done under the following boundary conditions.

$$\left.\begin{aligned} &r=a:\sigma_r^{\mathrm{II}}=\tau_{rz}^{\mathrm{II}}=0\\ &z_2=\pm h_2:u^{\mathrm{II}}=0,\quad \frac{\partial w^{\mathrm{II}}}{\partial r}=0\\ &\int_0^a 2\pi r(\sigma_z^{\mathrm{II}})_{z=h_2}\,dr=\int_0^a 2\pi rF(r)\,dr \end{aligned}\right\}\tag{14}$$

In numerical calculations, a tensile load $F(r)$ was assumed to act uniformly within

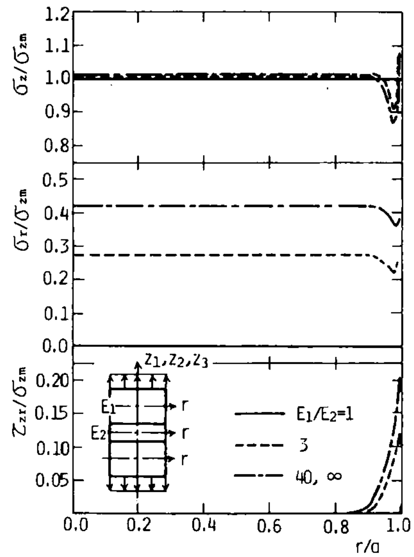

FIGURE 3 Effect of the ratio of Young's modulus on stress distribution at the interface ($z_2 = h_2$, $h_1/h_2 = 10$, $h_1/a = 0.2$, $\nu_1/\nu_2 = 1.0$, $F(r)$ is constant within the region $r \leqq a$.)

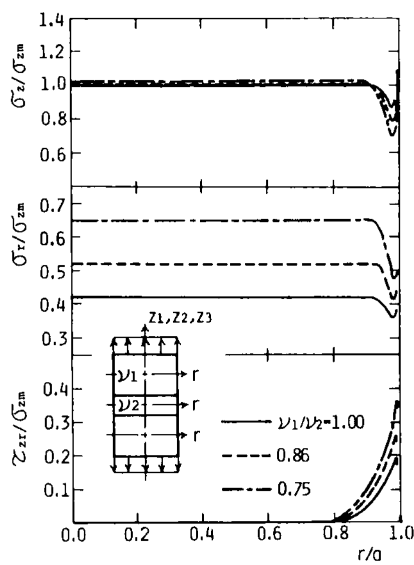

FIGURE 4 Effect of the ratio of Poisson's ratio on stress distribution at the interface ($z_2 = h_2$, $h_1/h_2 = 10$, $h_1/a = 0.2$, $E_1/E_2 = 40$, $F(r)$ is constant within the region $r \leqq a$.)

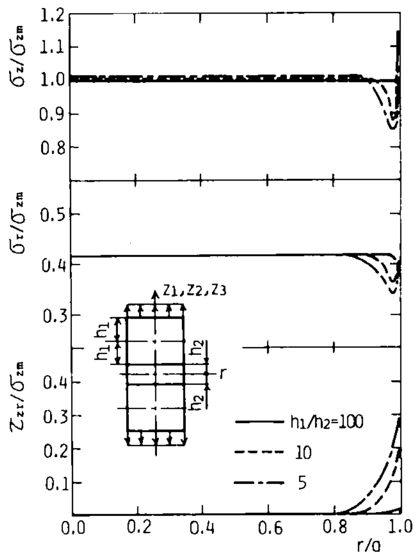

FIGURE 5 Effect of the thickness of adhesive on stress distribution ($z_2 = h_2$, $h_1/a = 0.2$, $E_1/E_2 = 40$, $\nu_1/\nu_2 = 1.0$, $F(r)$ is constant within the region $r \leqq a$.)

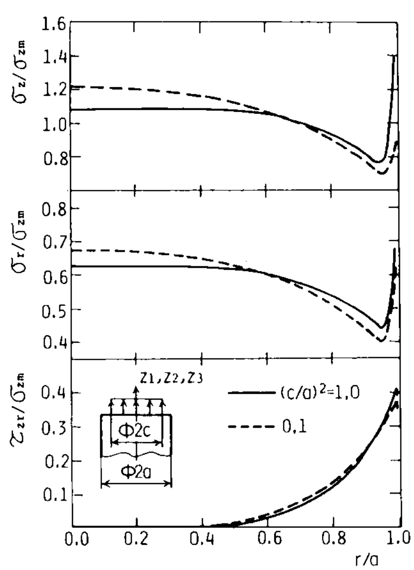

FIGURE 6 Effect of external load distribution on stress distribution at the interface ($z_2 = h_2$, $h_1/h_2 = 5$, $h_2/a = 0.1$, $E_1/E_2 = 65.6$, $\nu_1/\nu_2 = 0.81$.)

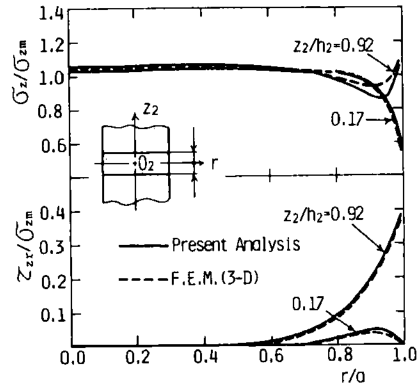

FIGURE 7 Comparison of analytical results with results obtained by F.E.M.[8] ($h_1/h_2 = 10$, $h_1/a = 1.0$, $E_1/E_2 = 91.0$, $\nu_1/\nu_2 = 0.91$, $F(r)$ is constant within the region $r \leqq a$.)

the region $r \leqq a$ on the upper surface of adherend ($z_1 = h_1$). In this figure, σ_{zm} represents the mean normal stress. The ordinate represents the ratios σ_z/σ_{zm}, σ_r/σ_{zm} of the normal stresses to the mean normal stress and τ_{zr}/σ_{zm} of the shear stress to the mean normal stress. The abscissa represents the ratio r/a of the distance r from the center to the radius a. It is seen that the distribution of σ_z tends to be averaged when the value E_1/E_2 approaches 1 and the singularity increases with an increase of the value E_1/E_2. It is also seen that the shear stress τ_{zr} near the edge $r/a = 1.0$ increases with an increase of the value E_1/E_2. The difference is not found among the distribution of σ_z in the cases where E_1/E_2 is 40, 60 and infinity, and neither is the distribution of τ_{zr}. In this model for analysis, adherends are assumed to be rigid when the value of E_1/E_2 is more than 40. Since the stress singularity is caused near the point of $r/a = 1.0$ at the interface, each stress is shown within the region $r/a = 0.99$ in the figures.

Figure 4 shows the effect of Poisson's ratio ν_1/ν_2 of adherends to that of the adhesive on the stress distributions σ_z, σ_r and τ_{zr} at the interface ($z_2 = h_2$). The variation of the distribution σ_z and the shear stress τ_{zr} decrease near the edge $r/a = 1.0$ when the ratio of Poisson's ratio ν_1/ν_2 approaches 1.

Figure 5 shows the effect of the thickness of adhesive on the stress distributions σ_z, σ_r and τ_{zr} at the interface ($z_2 = h_2$). The height $2h_1$ of adherends is held constant and the value of h_1/h_2 is varied as 5, 10, 20 and 100. Differences of the distributions σ_z and τ_{zr} are not found between the cases where h_1/h_2 is 20 and 100. The distribution of σ_z tends to be averaged and the shear stress τ_{zr} decreases with an increase of h_1/h_2.

Figure 6 shows the effect of the load distribution $F(r)$ on the stress distributions σ_z, σ_r and τ_{zr} at the interface. With the thickness of the adhesive held constant and the value of h_1/h_2 set at 5, computations were done in the cases where the uniform load $F(r)$ acts on the region $(c/a)^2 = 1.0$ and the region $(c/a)^2 = 0.1$. There is a visible effect of the load distribution on the stress distribution σ_z but very little effect on the distribution τ_{zr}. Next, computations were done with the value of h_1/h_2 set at 10. It was found that the effect of the load distribution was very little on the distributions of σ_z and τ_{zr}. From above results, it appears that there is very little effect on the stress distributions when the value of h_1/h_2 becomes more than 10 in this model for analysis. Figure 7 shows the comparison of the analytical results obtained by this study with the results obtained by F.E.M.[8] with respect to the stress distributions on the adhesive. The results obtained by F.E.M.[8] show the stresses at the centroid of the triangle elements. Therefore, the stresses obtained by the analysis were compared with those obtained by F.E.M. at $z_2/h_2=0.92$ and 0.17. Both results are in fairly good agreement.

4 CONCLUSIONS

This paper deals with a three-dimensional stress analysis of adhesive butt joints, in which two similar solid cylinders are joined by an adhesive and subjected to external tensile loads. The following results were obtained.

1) Replacing adherends and an adhesive with finite solid cylinders, respectively, a method of analysis of the stress distributions on joints is demonstrated as a three-body contact problem using the three-dimensional theory of elasticity.

2) Based on the method 1), the effects of the ratio of Young's modulus of adherends to that of an adhesive, the ratio of Poisson's ratios, the thickness of the adhesive, and external load distributions, on the stress distributions at the interface are clarified by numerical computations.

3) The analytical results obtained by the method mentioned in 1) are compared with the results obtained by F.E.M. with respect to the stress distributions on an adhesive. It is seen that they are in fairly good agreement.

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